USN | | | | | | | | | | |

MATDIP401

Fourth Semester B.E. Degree Examination, Dec.2016/Jan.2017
Advanced Mathematics – II

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Find the angle between any two diagonals of a cube. **(06 Marks)**

b. The direction cosines of three mutually perpendicular lines are l_1, m_1, n_1 l_2, m_2, n_2 and l_3, m_3, n_3. Show that the line with direction cosines $l_1 + l_2 + l_3$, $m_1 + m_2 + m_3$, $n_1 + n_2 + n_3$ is equally inclined to the above lines. **(07 Marks)**

c. Find the equations of the plane passing through the points (1, 2, 3) (0, 1, 4) and (0, 0, 1). **(07 Marks)**

2 a. Derive the equation to the plane in the intercept form $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$. **(06 Marks)**

b. Find the angle between the lines $\frac{x-1}{1} = \frac{y-5}{0} = \frac{z+1}{2}$ and $\frac{x+3}{3} = \frac{y}{5} = \frac{z-5}{2}$. **(07 Marks)**

c. Find the image of the point (1, 2, 3) in the line $\frac{x+1}{2} = \frac{y-3}{3} = -z$. **(07 Marks)**

3 a. Show that the position vectors of the vertices of a triangle $2i - j + k$, $i - 3j - 5k$, $3i - 4j - 4k$ form a right angled triangle. **(06 Marks)**

b. Find a vector of magnitude 12 units which is perpendicular to the vectors $\vec{a} = 4i - j + 3k$ and $\vec{b} = -2i + j - 2k$. **(07 Marks)**

c. Find λ so that the points $A(-1, 4, -3)$, $B(3, 2, -5)$, $C(-3, 8, -5)$ and $D(-3, \lambda, 1)$ are coplanar. **(07 Marks)**

4 a. Find the unit tangent vector of the space curve $x = 1 + t^3$, $y = 2t^3$, $z = 2 - t^3$ at $t = 1$. **(06 Marks)**

b. Find the angle between the tangents to the curve $\vec{r} = \left(t - \frac{t^2}{2}\right)i + t^2 j + \left(t + \frac{t^2}{2}\right)k$ at $t = \pm 1$. **(07 Marks)**

c. A particle moves along the curve whose parametric equations are $x = t - \frac{t^3}{3}$, $y = t^2$ and $z = t + \frac{t^3}{3}$, where 't' is the time. Find the velocity and acceleration at any time 't'. Also find their magnitudes at t = 3. **(07 Marks)**

5 a. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $x = z^2 + y^2 - 3$ at $(2, -1, 2)$. **(06 Marks)**

b. Find the constants a, b, c such that the vector,
$\vec{F} = (x + y + az)i + (bx + 2y - z)j + (x + cy + 2z)k$ is irrotational. **(07 Marks)**

c. If $\vec{A} = \text{grad}(x^3 + y^3 + z^3 - 3xyz)$ then find $\text{div}\,\vec{A}$ and $\text{curl}\,\vec{A}$. **(07 Marks)**

6	a.	Find the expression for $L[\sin at]$.	**(05 Marks)**
	b.	Find $L[t \sin at]$.	**(05 Marks)**
	c.	Find $L\left[\frac{1-e^{at}}{t}\right]$.	**(05 Marks)**
	d.	Find $L\left[e^{t} \cos^{2} 2t\right]$.	**(05 Marks)**
7	a.	Find $L^{-1}\left[\frac{s}{(s+2)(s^{2}+1)}\right]$.	**(06 Marks)**
	b.	Find $L^{-1}\left[\frac{s+2}{s^{2}+2s+2}\right]$.	**(07 Marks)**
	c.	Find $L^{-1}\left[\log\left[\frac{s^{2}+1}{s(s-1)}\right]\right]$.	**(07 Marks)**
8	a.	Using Laplace transform solve: $y'' - 2y' + y = e^{2t}$ with $y(0) = 0$ and $y'(0) = 1$.	**(10 Marks)**
	b.	Solve using Laplace transformation, method $y'' + 2y' - 3y = \sin t$, $y(0) = y'(0) = 0$.	**(10 Marks)**

* * * * *